A Novel Approach to Modeling the Nonlinear Propagation Characteristics of HTS Planar Transmission Lines

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ABSTRACT

The paper presents a model for characterizing non-linear effects in High Temperature Superconductor (HTS) transmission lines. The model is based on a recursive time domain approach. The changes in both the propagation and attenuation constants due to nonlinear effects are included in the analysis. The model accurately predicts the variation of the third order intercept (TOI) with respect to line width and length. The validity of the model has been verified by comparison with experimental results.

I. INTRODUCTION

Wave propagation along High Temperature Superconductor (HTS) planar transmission lines leads to the generation of harmonics as well as non-linear propagation effects at power levels approaching the critical value. The non-linear propagation effects include changes in the attenuation and phase shift along an HTS transmission line. The attenuation constant increases with power as a result of increasing ohmic losses [1]. This effect arises from an increase in the normal electron density and a decrease in the density of super electrons [1]. A shift in phase velocity is the result of a change in the stored kinetic energy in the HTS material [1]. This change in phase velocity corresponds to a variation of the propagation constant β of the transmission line.

Several papers have been published that propose models which characterize the nonlinear effects of HTS transmission lines [1]-[5]. Non-linear propagation effects have been characterized with respect to the power level based on the Ginzburg-Landau Theory using Finite-Difference Time Domain (FDTD) Techniques [1]. Although this method is accurate, it is very cumbersome to implement. A recent paper by Vendik, *et al*. [2] characterizes the increased attenuation due to non-linear effects, but does not account for variations in the kinetic inductance. To our knowledge, no model has yet been presented to predict the non-linear propagation characteristics of HTS transmission lines with respect to both the length and width.

We present in this paper a comprehensive model for the characterization of the non-linear effects with respect to harmonic generation, phase and attenuation along HTS transmission lines of varying lengths and widths. Our model uses a novel approach to predict a nonlinear change in the electrical length and attenuation by observing the generation of harmonics along the HTS line. For this model, the transmission line is divided into small subsections where the harmonics can be accurately analyzed. The harmonics then build progressively through the cascaded non-linear subsections until reaching the end of the transmission line. The model accurately predicts the variation of the third order intercept point with the line width and length The theoretical results achieved using this model are in good agreement with the measured data given in [3] for HTS coplanar lines having different line widths and lengths.

II. NONLINEAR MODEL

Consider the HTS transmission line shown in Figure 1. The line is divided into N cascaded subsections each of length Δx. The propagation and attenuation constants vary with the instantaneous level of voltage along the transmission line. We assume that the phase shift and attenuation along a single subsection are constant values $\beta\Delta x$ and $\alpha\Delta x$ for a given instant in time.

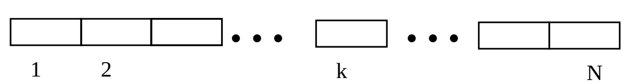

Figure 1. Division of the HTS planar transmission line into N subsections.

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In [1], a nonlinear full-wave solution based on Ginzburg-Landau (GL) theory has been developed using the Finite-Difference Time Domain (FDTD) technique. The theoretical results given in [1] show that the propagation constant β and attenuation constant α of HTS transmission lines vary linearly with applied power at the levels that are typically used for third order intercept measurements. The propagation and attenuation constants along each subsection can thus be approximated by:

$$\beta = \beta_0 \left(1 + \frac{a_1}{w^2} |V|^2 \right), \quad (1)$$

$$\alpha = \alpha_0 \left(1 + \frac{a_2}{w^2} |V|^2 \right), \quad (2)$$

where βo and α_0 are respectively the low power propagation and attenuation constants of the HTS transmission line. The value w is the transmission line width in μm. The constants a_1 and a_2 are functions of the material characteristics and are to be evaluated by fitting the TOI curves generated from the proposed model to measured data.

For the kth subsection at a given instant in time, the voltage can be written as

$$V_k = V_{k-1} \cos(\omega t - \beta_k \Delta x) e^{-\alpha_k \Delta x}, \quad (3)$$

Where β_k and α_k are given by:

$$\beta_k = \beta_0 \left[1 + \frac{a_1}{w^2} \left| V_{k-1} \cos(\omega t - \beta_{k-1} \Delta x) e^{-\alpha_{k-1} \Delta x} \right|^2 \right] \quad (4)$$

$$\alpha_k = \alpha_0 \left[1 + \frac{a_2}{w^2} \left| V_{k-1} \cos(\omega t - \beta_{k-1} \Delta x) e^{-\alpha_{k-1} \Delta x} \right|^2 \right]. \quad (5)$$

The signal components at the fundamental frequency and harmonics are calculated at the output of each subsection by taking a complex FFT of the time domain waveform (3). This signal is then applied to the next nonlinear subsection leading to the generation of a new set of harmonics. The level of the harmonics then builds progressively through each cascaded subsection until reaching the end of the transmission line. Figure 2 illustrates the concept of the proposed model.

III. COMPARISON WITH EXPERIMENTAL RESULTS

The data used to determine the validity of our nonlinear propagation model were published by Wilker, Shen, *et al*. in [3]. The measured data are for single tone TOI of a series of 50 ohm TBCCO coplanar transmission lines of varying widths and lengths. The substrate used is $LaAlO_3$ with a value $\varepsilon_r = 24$ and $\tan\delta = 10^{-5}$. A stimulus frequency of 1.5 GHz was used in simulating the transmission line. The input power was stepped from 0 dBm to 40 dBm. The constants a_1 and a_2 were set to the values corresponding to the temperature, and the length of the line was varied in accordance with the measured results. The constants a_1 and a_2 are only valid for a single temperature since the nonlinear effects increase as the temperature approaches the critical value. The optimized values for these constants at 90K and 100K are given in Table I. As expected, the nonlinear effects are more pronounced at 100K, as indicated by the higher constant values.

According to Megahed *et al*. [1], the nonlinear effect of the attenuation constant is greater than for the phase velocity (and thus β). This difference arises since the

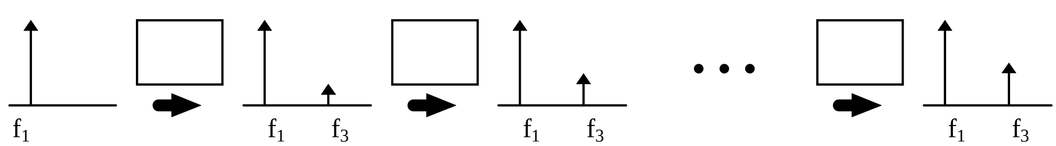

Figure 2. The third harmonic level increases as the signal propagates through each segment of the HTS planar transmission line.

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ohmic attenuation effect is greater than the change in stored kinetic energy that causes the shift in phase velocity [1]. The optimized constants are consistent with the results of Megahed, *et al*. As seen in Table I, the attenuation model constant a_2 is higher than that of the phase model constant a_1.

TABLE I Optimized Values of the Nonlinear Propagation Model Constants a_1 and a_2

T(K)	**Optimized a_1**	**Optimized a_2**
90	0.0148	0.7
100	0.58	12.85

Figure 3 illustrates the third order intercept values for different line widths. The results achieved are consistent with expectations since the current density is lower for a wider line. The current density is thus farther from the critical level and the nonlinear effect decreases. Figure 4 indicates that an increase in line length yields a lower third order intercept point. This was also expected since the signal propagates along a greater distance through the HTS line, augmenting the nonlinear effect.

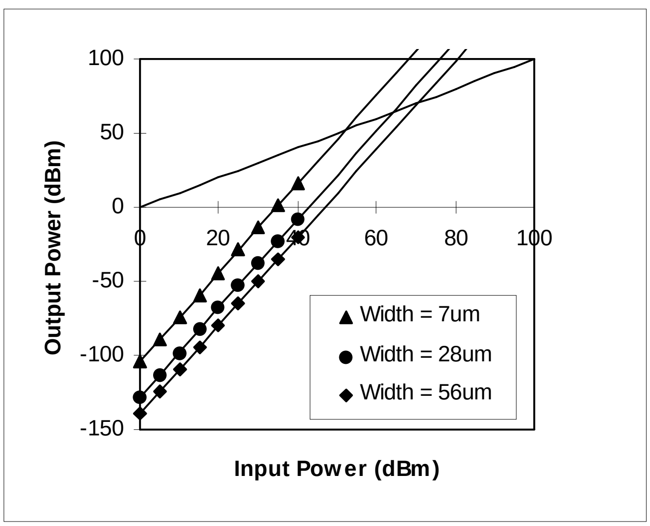

Figure 3. The fundamental and third harmonic output power vs. input power for line widths of 7µm, 28µm and 56µm at 90K. The points represent the third harmonic output levels calculated by the nonlinear model. The lines of best fit were extended to calculate the third order intercept value.

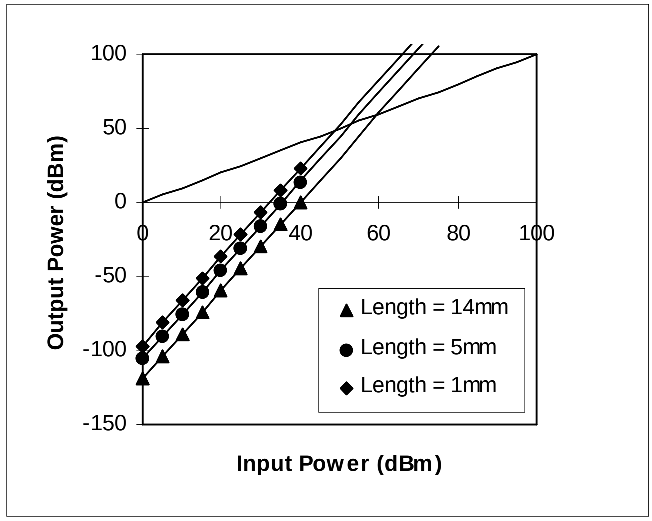

Figure 4. The fundamental and third harmonic output power vs. input power for line lengths of 14mm, 5mm and 1mm at 100K.

The calculated values of the TOI points correlated very well with the published results from Wilker, Shen, *et al* [3]. As shown in Tables II and III. The model accurately predicts the variations of the third order intercept with respect to both line width and length. The data point for a 1mm line at 90K has an unknown measurement tolerance level [3]. The calculated value differed from the measured mean by 4% which may be the result of greater measurement error. The other values corresponded well to the measured results, differing between 0.19% and 1.8% from the measured mean TOI level.

TABLE II Comparison of Measured [3] vs. Calculated TOI (dB) for Varying Line Widths

T (K)	**Width (µm)**	**Measured TOI (dB)**	**Calculated TOI (dB)**	**% Deviation from Measured Mean**
90	56	71.2±0.9	70.3	1.3%
90	28	64.5±0.6	64.1	0.62%
90	7	51.3±0.7	52.0	1.4%
100	56	55.4±0.3	54.4	1.8%
100	28	48.5±0.4	48.4	0.21%
100	7	37.3±0.4	37.4	0.27%

TABLE III Comparison of Measured [3] vs. Calculated TOI (dB) for Varying Line Lengths

T (K)	Length (mm)	Measured TOI (dB)	Calculated TOI (dB)	% Deviation from Measured Mean
90	14	64.5±0.6	64.1	0.62%
90	5	68.6±1.1	69.0	0.58%
90	1	75.0±?	78.1	4.0
100	14	48.5±0.4	48.4	0.21%
100	5	52.8±0.9	52.9	0.19%
100	1	60.4±2.4	59.9	0.83%

The constants a_1 and a_2 in equations (1) and (2) were set to the values corresponding to the temperature, and the length of the line was varied in accordance with the measured results. The change in the third order intercept due to varying the length correlated well with measured results.

IV. CONCLUSION

A nonlinear propagation model for high temperature superconducting planar transmission lines has been presented. The nonlinear model accurately predicts the third order intercept over varying line lengths and widths. The theoretical results achieved are in good agreement with measured data (within 2 %). Our model, when fitted to measured results, can be used to predict the non-linear propagation effects at varying power levels at a given temperature. Further work is needed to include the temperature effects in the nonlinear model.

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